

herm1t : Multiplicative PRIDE

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Multiplicative PRIDE

Last time, I explored [how PRIDE works](#). It turned out that the initial value and the generator in the group Z_n under addition are simply split into two variables. Can we do something similar with multiplication? Let's try the following example:

[pride-mul.c](#)

```
for (a = 1; a < n; a++)
for (b = 1; b < n; b++) {
    /* GCD(b, n)          A062955 phi(n^2) - phi(n)
       GCD(a * b, n)     A127473 phi(n)^2 */
    if (gcd(b, n) != 1)
        continue;
    bzero(o, sizeof(o));
    for (y = 0, i = 0; i < n; i++) {
        /* x = (a + b * i) % n; */
        x = (y + a) % n;
        y = (y + b) % n;
    }
}
```

Although it looks very similar to the previous one, this is not group addition, but an affine transformation in the *ring* Z_n , of the form $x = a + b * i$, where $\text{GCD}(b, n) == 1$.

If you calculate the number of possible permutations for different n , you get sequence [A062955](#) in OEIS:

```
for a in `seq 1 16`; do ./a.out $a|sort|uniq|wc -l; done|xargs
0 1 4 6 16 10 36 28 48 36 100 44 144 78 112 120
```

That is: $\phi(n^2) - \phi(n) = (n-1) \times \phi(n)$, where ϕ is Euler's totient function.